

Helioseismic Perspective of the Solar Dynamo

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Abstract. Helioseismology has been, without a doubt, one of the greatest contributors to our understanding of the solar cycle. In particular, its results have been critical in the development of solar dynamo models, by providing modelers with detailed information about the internal, large scale flows of solar plasma.

This review will give a historical overview of the evolution of our understanding of the solar cycle, placing special emphasis on advances driven by helioseismic results. We will discuss some of the outstanding modeling issues, and discuss how Helioseismology can help push our understanding forward during the next decade.

1. Introduction

Fifty years have passed since the discovery of the 5-minute oscillations by Leighton, Noyes, & Simon (1962), and helioseismology has come a long way since then. As a person whose first contact with solar physics was only 10 years ago, it is very hard for me to imagine a time where we did not know that the Sun oscillates at specific frequencies, and that these can be used to probe the interior of the Sun. As a dynamo theorist it is also very hard to imagine how our understanding of the solar cycle would have advanced (if at all) without helioseismic measurements. It also is incredible to realize that the birth of solar dynamo theory had only taken place a few years before the discovery of the 5-minute oscillations, and that Leighton himself was one of the main contributors to the picture of the solar cycle that we have today.

Although by no means comprehensive, this brief review is meant as a tribute to those that were there from the beginning and those who have come along the way. Congratulations to all helioseismologists for their immeasurable contribution to the progress and prominence of solar physics, and for pushing the limits of our models. Thanks to all of you dynamo theory growers and prosperous, as it meets the challenges inherent with trying to match understanding with nature.

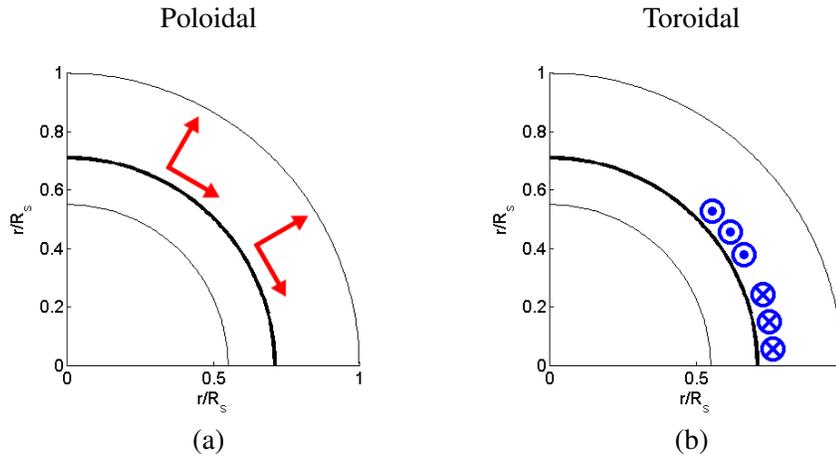


Figure 1. Important terminology: (a) Poloidal components are confined to the meridional plane, B_r and B_θ . (b) The toroidal component is normal to the meridional plane, B_ϕ (i. e., in the direction of rotation).

2. Evolution of our Understanding of the Solar Cycle: A Path Heavily Influenced by Helioseismology

In a nutshell, the solar magnetic cycle is seen as a process in which the magnetic field switches from a configuration which is predominantly poloidal (confined to the meridional plane, B_r and B_θ ; see Figure 1a) to one which is predominantly toroidal (normal to the meridional plane, B_ϕ ; see Figure 1b) and back, drawing on the available energy in solar plasma flows.

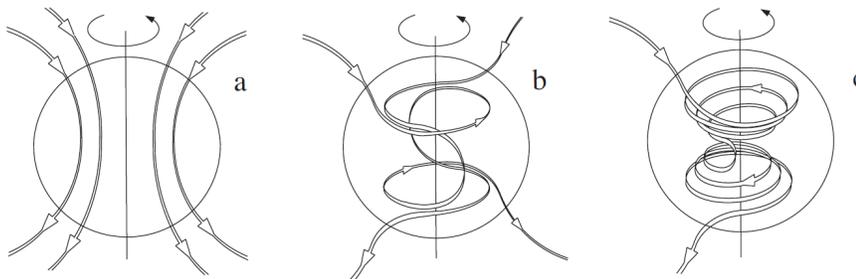


Figure 2. Poloidal \rightarrow Toroidal field conversion. The interaction of a predominantly poloidal field (a) with differential rotation builds up a toroidal component (b), which given enough time produces a predominantly toroidal configuration (c). Illustrations courtesy of J. J. Love

The first part of the process (Poloidal \rightarrow Toroidal field) was proposed in the context of the Sun by (Larmor 1919) and relies on the fact that the Sun does not rotate uniformly. The idea is that the shearing of a large scale poloidal field by the solar dif-

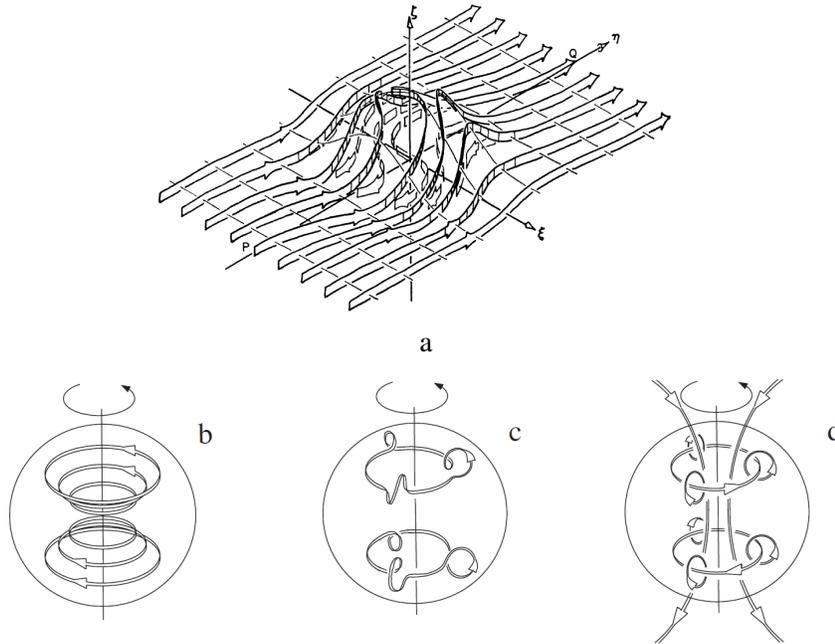


Figure 3. Toroidal \rightarrow Poloidal field conversion. The interaction between the toroidal field (b) and helical turbulence can impart a poloidal component to the field (a). When this happens at a global scale (c), the resulting configuration acquires a poloidal component which closes the cycle setting up the stage for the next one (d). Illustrations (a) by Parker (1955) reproduced by permission of the AAS and (c–d) by courtesy of J. J. Love

ferential rotation results in the production of large scale toroidal belts of opposite sign across the equator. These toroidal belts then act as the source of active regions which, due to this antisymmetry across the equator, match Hale's law. This process, beautifully illustrated in Figure 2, is very well established in our understanding of the solar magnetic cycle. It is the mechanism behind the second part (Toroidal \rightarrow Poloidal) which has proven elusive and is still debated.

The first breakthrough in this direction was made by Parker (1955). His idea was that the effect of the Coriolis force on turbulent convection could impart a systematic twist to toroidal fields producing a net poloidal component (Figure 3a); the global effect of this small scale dynamo would work together to produce a global poloidal field closing the cycle (Figure 3b–c). Furthermore, Parker found that solutions of such a system consisted of propagating waves, which signify the migration of active latitudes with the progress of the solar cycle.

Another possible explanation was proposed by Babcock (1961) and further elaborated by Leighton (1964, 1969); envisaged as a shallow dynamo operating in the near surface layers of the Sun, as opposed to the mean-field dynamo (proposed by Parker), which operates throughout the solar convection zone. As with Parker's idea, the first part of the cycle (Poloidal \rightarrow Toroidal) is achieved by the shearing of the poloidal field by differential rotation. However, the second part of the cycle (Toroidal \rightarrow Poloidal),

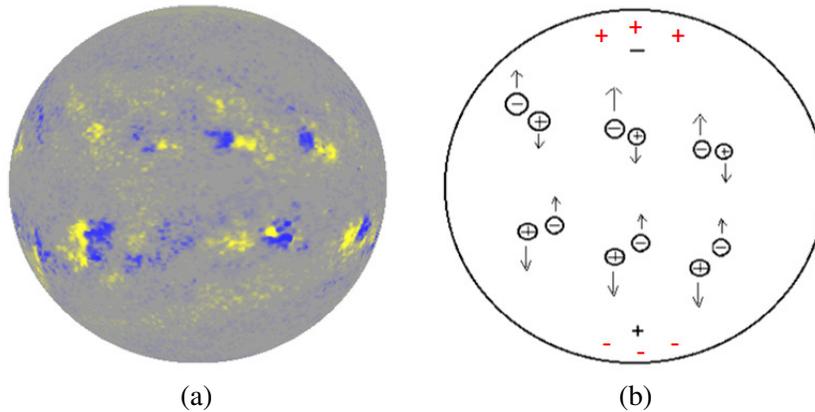


Figure 4. (a) MDI magnetogram showing a snapshot of photospheric magnetic field. The systematic orientation and tilt known as the Hale’s and Joy’s laws can be seen very clearly: In the northern (southern) hemisphere the leading (eastmost) polarity is consistently positive (negative), colored in yellow (blue), and it is closer to the equator than the following (westmost) polarity of the opposite sign. (b) Simplified diagram of this typical configuration, showing the form of polarity migration that leads to flux cancellation across the equator and accumulation at the poles. This accumulation of field cancels and then reverses the old polarity (shown in red).

is achieved by the collective effect of bipolar active region emergence and decay. At the core of this process, known as the Babcock–Leighton (BL) mechanism, resides the fact that active regions have a systematic hemispheric orientation and tilt (Hale’s and Joy’s laws). This means that the leading polarity of most active regions is closer to the equator than the following polarity. Given that this orientation is opposite in each hemisphere, there is a net cancellation of flux across the equator and a net accumulation of open field at the poles, which produces the cancellation and reversal of the poloidal field closing the cycle (see Figure 4). Another way of understanding this process is in terms of the magnetic moment: due to their systematic orientation and tilt, most active regions in a cycle will carry a dipole moment of the same sign (and of opposite sign as that of the old cycle’s dipole moment). After eleven years of active region emergence and diffusive action, higher-order moments would have decayed leaving a new bipolar field as the starting point for the next cycle. It is important to note that, at the time, Parker’s idea of a turbulent source of poloidal field was widely accepted (while the BL mechanism was not). It would not be until recent times that the BL mechanism would be resurrected and transformed into what it is today (see below).

2.1. The Propagation of the Dynamo Wave and the Measurement of the Differential Rotation by Helioseismology

Parker, in his landmark paper (1955), found that the linear dynamo equations support traveling wave solutions; which was found to be also true for spherical coordinates and non-linear models (Yoshimura 1975; Stix 1976). The direction of propagation of such waves, s , was found to be:

$$s = \alpha \vec{\nabla} \Omega \times \hat{e}_\phi, \quad (1)$$

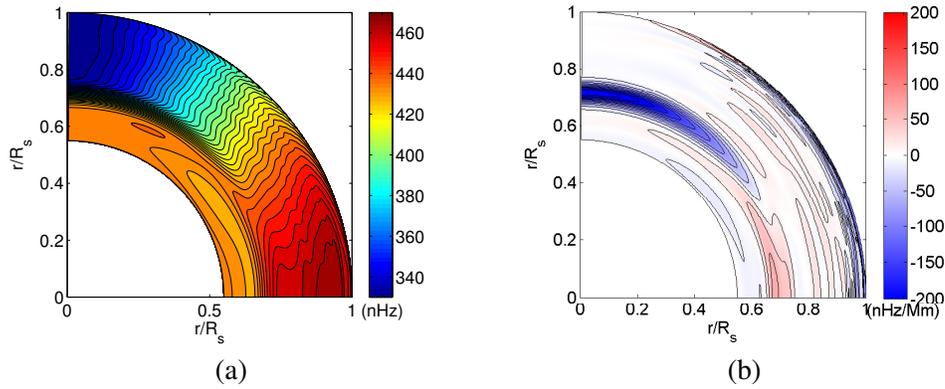


Figure 5. (a) Differential rotation composite used in Muñoz-Jaramillo et al. 2009. It is constructed using a combination of data from the Global Oscillation Network Group obtained using the RLS inversion (courtesy R. Howe), and the analytical form of Charbonneau et al. (1999) (b) Radial shear of the composite differential rotation profile.

where Ω quantifies the solar differential rotation, which means that in order to have equatorial propagation of the dynamo wave, as observed in active latitudes, the following condition must be satisfied:

$$\alpha \frac{\partial \Omega}{\partial r} < 0. \quad (2)$$

Given that at that time the shape of the differential rotation was unknown, there was relative freedom regarding its shape inside the convection zone (and modelers assumed shear was negative since α is positive). However, once the differential rotation was measured accurately (Thompson et al. 1996; Kosovichev et al. 1997; Schou et al. 1998), the correct profile turned out to have a positive radial shear at low latitudes (shown in Figure 5), leading to poleward propagating solutions (see Equation 2). This created a problem that needed to be addressed by dynamo theory.

2.2. The Arrival of Flux-Tube Simulations

Another blow to the accepted understanding of the solar dynamo came as a consequence of the development of simulations of the buoyant toroidal flux-tubes (resulting in the emergence of active regions at the surface). These simulations showed that only flux-tubes with a field strength of the order of $10^4 - 10^5$ Gauss are consistent with observed latitudes of emergence (first reported by Choudhuri & Gilman 1987) and observed active region tilt (first reported by D’Silva & Choudhuri 1993). These results have been confirmed by several independent studies of increasing sophistication (Choudhuri 1989; Fan et al. 1993, 1994; Schüssler et al. 1994; Caligari et al. 1995; Fan & Fisher 1996; Caligari et al. 1998; Fan & Gong 2000; Weber et al. 2011, 2013).

The conflict arises when one recalls that Parker’s proposed mechanism for poloidal field regeneration involves the twisting of the toroidal field by helical turbulent convection. In order to do this efficiently, the plasma needs to have enough energy to dominate the dynamics of the combined system and this mechanism is expected to saturate once the energy density of the magnetic field reaches equipartition values. The problem is

that the equipartition magnetic field B_e is two orders of magnitude below the values found to be necessary to form ARs (50 – 100 KGauss), making it quite difficult to reconcile dynamo theory with flux-tube simulations.

In spite of this setback, the dynamo community was quick to step up to the challenge and started looking for alternate sources of poloidal field regeneration. It is beyond the scope of this introduction to make a comprehensive review of all of them; for which we point the interested reader to the review by Charbonneau (2010). Instead, we will concentrate on what has become the most widely accepted mechanism for regenerating the poloidal field: the Babcock–Leighton mechanism.

2.3. The Discovery of the Meridional Flow and the Rise of Flux-Transport Dynamos

While flux-tube simulations were casting doubt on classical $\alpha\Omega$ dynamos, the development of high resolution magnetographs was paving the way for a discovery that would have far reaching consequences for dynamo models: the meridional circulation. Observations of small magnetic features on the surface of the Sun showed a 10 – 20 m/s flow of mass from the equator towards the poles (Komm et al. 1993; Latushko 1994; Snodgrass & Dailey 1996; Hathaway 1996). These observations were later confirmed by helioseismic measurements, which found that the poleward flow is present in at least the top 10% of the convection zone (Giles et al. 1997; Schou & Bogart 1998; Braun & Fan 1998; González Hernández et al. 1999). While the exact structure of the meridional flow in the bulk of the convection zone is still a subject of debate, it has been generally assumed to have a return flow somewhere deep in the convection zone.

From the point of view of dynamo theory, the existence of such a large scale flow has a very important implication: since the field on the Sun is frozen in the plasma (Alfvén 1942), an equatorward flow at the bottom of the convection zone will drag the field along – helping circumvent the condition required for an equatorial propagation of the dynamo wave obviating the requirement altogether (Equation 2; Choudhuri, Schussler, & Dikpati 1995; Durney 1995). Because of this, meridional circulation has become an integral part of modern kinematic dynamo models; giving rise to what is generally referred to as flux-transport dynamos. This type of model relies on a deep return flow to determine the direction of propagation of the dynamo wave and places the role of poloidal field generation on active region emergence and decay (BL mechanism). However, contrary to Babcock’s original idea, modern BL dynamo models do not operate as a shallow dynamo. This is because a shallow dynamo is incompatible with our current understanding of active regions as the top part of buoyant flux-tubes, which rise from the bottom of the convection zone. Instead, they take the best aspects of both types of dynamos by amplifying and storing the toroidal field inside the convection zone and recreating the poloidal field through the BL mechanism.

3. Helioseismic Frontiers from the Point of View of the Solar Dynamo

As was evident in the previous section, helioseismology is one of the main sources of constraints for solar dynamo models. Because of this, it is to be expected that any new developments have the potential of transforming our understanding of the solar cycle. Although by no means fully comprehensive, in this section we make a short overview of two helioseismic results that are going to contribute greatly to the improvement of our understanding of the solar dynamo during the next decade.

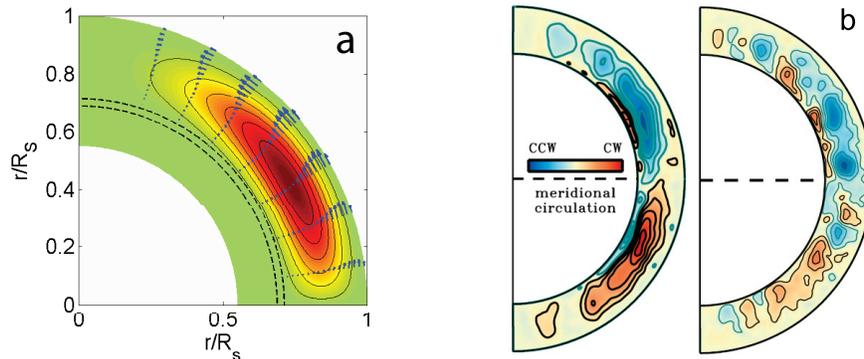


Figure 6. (a) Meridional flow profile used by Muñoz-Jaramillo et al. 2009. (b) Meridional flow profiles obtained using the ASH code (taken from Miesch et al. 2008). The left panel shows the streamlines of mass flux averaged over longitude and time (140 days). The right panel shows an instantaneous snapshot taken during that interval. Blue color and grey (red color and black) contours represent counter clockwise (clockwise) circulation. Panel (b) reproduced by permission of the AAS

3.1. Measurements of the Deep Meridional Flow

A crucial ingredient in modern dynamo models, meridional flow is believed to be responsible for setting the speed of the migrating toroidal belts at the bottom of the convection zone, from which sunspots emerge. Additionally, it plays an important role in setting the period and amplitude of the cycle, as well as affecting the amount of flux cancellation across the equator and flux accumulation at the poles. However, as mentioned in the previous section, it has only been measured with certainty in the top 10% of the convection zone.

Typical flux-transport dynamos incorporate meridional flow by assuming a single global cell (see Figure 6-a and review by Charbonneau 2010 and references therein). However, the meridional flow patterns seen to arise in anelastic magneto-hydrodynamic (MHD) simulations (see for example Miesch et al. 2008; Racine et al. 2011; Warnecke et al. 2013) show a highly fluctuating, and multicellular, large scale flow-pattern (see Figure 6-b). Recent attempts to measure the deep meridional flow, both using helioseismic (Zhao et al. 2013) and Doppler maps (Hathaway 2012), suggest that this is the case. However, a multicellular meridional flow changes the behavior of typical flux-transport dynamos; shortening the cycle period and potentially changing the parity of the preferred solutions (Jouve & Brun 2007).

As more evidence accumulates, it seems we are about to face a paradigm shift akin to the one caused by the measurement of the internal differential rotation. The most likely direction for the evolution for our theoretical understanding is the incorporation of the turbulent magnetic pumping caused by the transport of magnetic flux due to the presence of density and turbulence gradients in convectively unstable layers (see for example Dorch & Nordlund 2001; Ziegler & Rüdiger 2003). When translated into a mean field context, it has been found that the extracted turbulent pumping has an amplitude of the same order of magnitude as the poloidal source, which seems to play a

crucial role in the evolution of the large-scale magnetic field (Käpylä et al. 2006; Racine et al. 2011; Simard et al. 2013).

Translated into a mean-field context, turbulent pumping appears as an additional velocity field that has been shown to improve the agreement between the simulated and observed magnetic field (Guerrero & de Gouveia Dal Pino 2008). More importantly, it allows flux-transport dynamos to operate without the need for a deep meridional flow cell. Another interesting consequence of the introduction of turbulent pumping in kinematic dynamos is a reduction of solar cycle memory (see Karak & Nandy 2012) consistent with that found by observations ((see Muñoz-Jaramillo et al. 2013). Perhaps too crucial a role was being (mis)placed on the shoulders of the meridional flow, forcing it to absorb the brunt of solar cycle timing and most of the magnetic flux transport. It is likely that during the next decade we will see a paradigm shift where the most important mechanisms of flux transport are now turbulent in nature.

3.2. Seismic Constraints on Interior Solar Convection

One of the most important development of the last decade is the arrival, in force, of the global anelastic MHD simulations – see for example the ASH code (Clune et al. 1999) and the EULAG code (Smolarkiewicz & Charbonneau 2013). The number of interesting results and publications resulting from this type of simulation is staggering and is outside the scope of this paper; however, it is clear that they have become one of the most important tools for advancing our understanding of solar turbulent convection and its interaction with magnetic fields. Furthermore, it has been found that driving a kinematic mean-field simulation using turbulent coefficients, extracted from an anelastic MHD simulation (poloidal source and turbulent pumping), results in cycles in remarkable agreement with their MHD counterpart (Simard et al. 2013). This establishes a strong link between these two types of simulations and demonstrates the viability of anelastic MHD simulations as a source of constraints for kinematic dynamos.

As usually happens when observations tighten the screws on unconstrained parameters (as happened when helioseismology measured the solar internal rotation profile; see Section 2.1), theorists are forced to revisit their understanding and propose alternatives for the inconsistencies between their models and observations. As mentioned in the previous subsection, the mounting evidence suggesting that the meridional flow is highly variable and complex flow pattern is leading dynamo theory to shift into a new paradigm where the most important mechanisms of magnetic flux transport are turbulent in nature. This of course means that the attention of observers and theoreticians needs to focus on the issue of solar turbulent convection.

The beginning of this important conversation has already started through the pioneering work of Hanasoge et al. (2010, 2012) and Miesch et al. (2012). The basic idea is to place limits on the magnitude of the convective velocity by using helioseismic measurements (Hanasoge et al.) and theoretical considerations (Miesch et al.); see Figure 7. These limits can then be used to improve the fidelity of anelastic MHD simulations and the properties of their convection. Considering that the solar dynamo picture is shifting to place more importance on turbulent mechanisms of flux transport, and that anelastic MHD simulations can be used to constrain kinematic dynamos, it is very important that this kind of work continues. Not only that, it should ideally be performed by dynamo theorists and helioseismologists working together, and performed on more than one kind of anelastic MHD simulation.

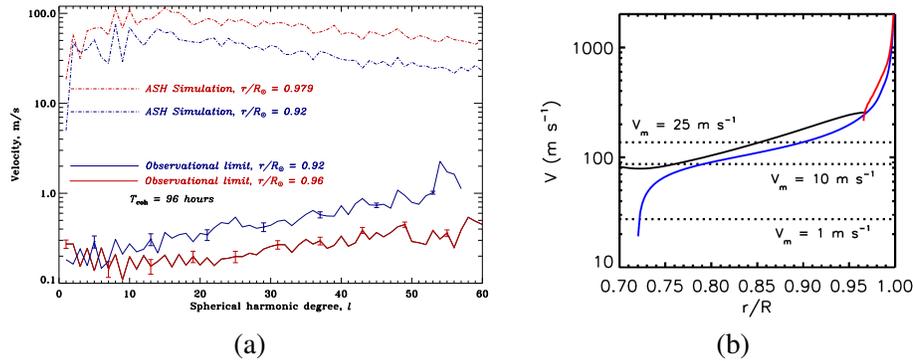


Figure 7. (a) Upper bounds on the velocity magnitude spectrum of interior convection (with differential rotation and meridional circulation removed), based on the sensitivity of helioseismic waves to large-scale convective cells (solid lines). Red and blue indicate two different focus depths. An equivalent spectrum for the convection in the ASH code is shown for comparison (dashed lines). Figure taken from (Hanasoge et al. 2012). Reproduced by permission of the National Academy of Sciences. (b) Comparison of theoretical lower limits for the convection amplitude (dotted lines) with numerical and theoretical models of convection (ASH \rightarrow solid black, MURAM \rightarrow solid red, and a combination of a mixing-length model and STAGGER \rightarrow solid blue). Figure taken from (Miesch et al. 2012). Reproduced by permission of the AAS

4. Concluding Remarks

It is well known that the evolution of our understanding of the dynamo is intertwined with achievements in the field of helioseismology (something that is evident all through this review). This symbiotic relationship is not likely to diminish anytime soon; however, the arrival of global anelastic MHD simulations has changed the landscape of this relationship forever.

Recent theoretical work indicates that the complexity shown by the latest measurements of the deep meridional flow is giving rise to a paradigm shift in our understanding (similar in magnitude as the one caused by the measurement of the internal rotation of the Sun); forcing us to shift our perspective of flux-transport from one dominated by the meridional flow to one dominated by turbulent mechanisms. Considering the strong connection demonstrated between kinematic and anelastic MHD simulations (see Simard et al. 2013), it is clear that a more intimate interaction between kinematic and anelastic MHD simulations will play a crucial role in the birth of this new understanding.

It is important to note that, as anelastic MHD simulations become a crucial source of constraints for turbulent mechanisms of flux transport in kinematic dynamos, it is of paramount importance that the properties of turbulent convection are captured by simulations as accurately as possible. This will only be possible through the interplay between helioseismic observations and theoretical considerations as demonstrated by the work of Hanasoge et al. (2010, 2012) and Miesch et al. (2012).

One thing is certain, now more than ever is necessary for dynamo theorists and helioseismologists to work together. If this collaboration is successful, it will usher in a new age in our understanding. Very exciting times are coming.

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References

- Alfvén, H. 1942, *Arkiv Astron.*, 28, 1
 Babcock, H. W. 1961, *ApJ*, 133, 572
 Braun, D. C., & Fan, Y. 1998, *ApJ*, 508, L105
 Caligari, P., Moreno-Insertis, F., & Schussler, M. 1995, *ApJ*, 441, 886
 Caligari, P., Schuessler, M., & Moreno-Insertis, F. 1998, *ApJ*, 502, 481
 Charbonneau, P. 2010, *Living Reviews in Solar Physics*, 7, 3
 Choudhuri, A. R. 1989, *Solar Phys.*, 123, 217
 Choudhuri, A. R., & Gilman, P. A. 1987, *ApJ*, 316, 788
 Choudhuri, A. R., Schussler, M., & Dikpati, M. 1995, *A&A*, 303, L29
 Clune, T., Elliott, J., Miesch, M., Toomre, J., & Glatzmaier, G. 1999, *Parallel Comput.*, 25, 361
 Dorch, S. B. F., & Nordlund, Å. 2001, *A&A*, 365, 562
 D’Silva, S., & Choudhuri, A. R. 1993, *A&A*, 272, 621
 Durney, B. R. 1995, *Solar Phys.*, 160, 213
 Fan, Y., & Fisher, G. H. 1996, *Solar Phys.*, 166, 17
 Fan, Y., Fisher, G. H., & Deluca, E. E. 1993, *ApJ*, 405, 390
 Fan, Y., Fisher, G. H., & McClymont, A. N. 1994, *ApJ*, 436, 907
 Fan, Y., & Gong, D. 2000, *Solar Phys.*, 192, 141
 Giles, P. M., Duvall, T. L., Scherrer, P. H., & Bogart, R. S. 1997, *Nature*, 390, 52
 González Hernández, I., Patrón, J., Bogart, R. S., & The SOI Ring Diagram Team 1999, *ApJ*, 510, L153
 Guerrero, G., & de Gouveia Dal Pino, E. M. 2008, *A&A*, 485, 267
 Hanasoge, S. M., Duvall, T. L., & Sreenivasan, K. R. 2012, *Proc. Nat. Acad. Sci.*, 109, 11928
 Hanasoge, S. M., Duvall, T. L., Jr., & DeRosa, M. L. 2010, *ApJ*, 712, L98
 Hathaway, D. H. 1996, *ApJ*, 460, 1027
 — 2012, *ApJ*, 760, 84
 Jouve, L., & Brun, A. S. 2007, *A&A*, 474, 239
 Käpylä, P. J., Korpi, M. J., Ossendrijver, M., & Stix, M. 2006, *A&A*, 455, 401
 Karak, B. B., & Nandy, D. 2012, *ApJ*, 761, L13
 Komm, R. W., Howard, R. F., & Harvey, J. W. 1993, *Solar Phys.*, 147, 207
 Kosovichev, A. G., Schou, J., Scherrer, P. H., Bogart, R. S., Bush, R. I., Hoeksema, J. T., Aloise, J., Bacon, L., Burnette, A., de Forest, C., Giles, P. M., Leibbrand, K., Nigam, R., Rubin, M., Scott, K., Williams, S. D., Basu, S., Christensen-Dalsgaard, J., Dappen, W., Rhodes, E. J., Jr., Duvall, T. L., Jr., Howe, R., Thompson, M. J., Gough, D. O., Sekii, T., Toomre, J., Tarbell, T. D., Title, A. M., Mathur, D., Morrison, M., Saba, J. L. R., Wolfson, C. J., Zayer, I., & Milford, P. N. 1997, *Solar Phys.*, 170, 43
 Larmor, J. 1919, *Brit. Assn. Adv. Sci. Rep.*, 159
 Latushko, S. 1994, *Solar Phys.*, 149, 231
 Leighton, R. B. 1964, *ApJ*, 140, 1547
 — 1969, *ApJ*, 156, 1
 Leighton, R. B., Noyes, R. W., & Simon, G. W. 1962, *ApJ*, 135, 474
 Miesch, M. S., Brun, A. S., De Rosa, M. L., & Toomre, J. 2008, *ApJ*, 673, 557
 Miesch, M. S., Featherstone, N. A., Rempel, M., & Trampedach, R. 2012, *ApJ*, 757, 128
 Muñoz-Jaramillo, A., Dasi-Espuig, M., Balmaceda, L. A., & DeLuca, E. E. 2013, *ApJ*, 767, L25

- Parker, E. N. 1955, *ApJ*, 122, 293
- Racine, É., Charbonneau, P., Ghizaru, M., Bouchat, A., & Smolarkiewicz, P. K. 2011, *ApJ*, 735, 46
- Schou, J., Antia, H. M., Basu, S., Bogart, R. S., Bush, R. I., Chitre, S. M., Christensen-Dalsgaard, J., di Mauro, M. P., Dziembowski, W. A., Eff-Darwich, A., Gough, D. O., Haber, D. A., Hoeksema, J. T., Howe, R., Korzennik, S. G., Kosovichev, A. G., Larsen, R. M., Pijpers, F. P., Scherrer, P. H., Sekii, T., Tarbell, T. D., Title, A. M., Thompson, M. J., & Toomre, J. 1998, *ApJ*, 505, 390
- Schou, J., & Bogart, R. S. 1998, *ApJ*, 504, L131
- Schüssler, M., Caligari, P., Ferriz-Mas, A., & Moreno-Insertis, F. 1994, *A&A*, 281, L69
- Simard, C., Charbonneau, P., & Bouchat, A. 2013, *ApJ*, 768, 16
- Smolarkiewicz, P. K., & Charbonneau, P. 2013, *J. Comput. Phys.*, 236, 608
- Snodgrass, H. B., & Dailey, S. B. 1996, *Solar Phys.*, 163, 21
- Stix, M. 1976, *A&A*, 47, 243
- Thompson, M. J., Toomre, J., Anderson, E. R., Antia, H. M., Berthomieu, G., Burtonclay, D., Chitre, S. M., Christensen-Dalsgaard, J., Corbard, T., De Rosa, M., Genovese, C. R., Gough, D. O., Haber, D. A., Harvey, J. W., Hill, F., Howe, R., Korzennik, S. G., Kosovichev, A. G., Leibacher, J. W., Pijpers, F. P., Provost, J., Rhodes, E. J., Jr., Schou, J., Sekii, T., Stark, P. B., & Wilson, P. R. 1996, *Science*, 272, 1300
- Warnecke, J., Käpylä, P. J., Mantere, M. J., & Brandenburg, A. 2013, *ArXiv e-prints*. 1301.2248
- Weber, M. A., Fan, Y., & Miesch, M. S. 2011, *ApJ*, 741, 11
- 2013, *Solar Phys.*, 287, 239
- Yoshimura, H. 1975, *ApJ*, 201, 740
- Zhao, J., Bogart, R. S., Kosovichev, A. G., Duvall, T. L., Jr., & Hartlep, T. 2013, *ApJ*, 774, L29
- Ziegler, U., & Rüdiger, G. 2003, *A&A*, 401, 433